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EFFECT OF SHAPED-CHARGE JET COMPRESSIBILITY AND STRENGTH ON THE CHARACTERISTICS OF THEIR INERTIAL STRETCHING IN FREE FLIGHT

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In [1], the authors considered a physicomathematical model of deformation of an element of a shaped-charge jet (SCJ) at the stage of uniform stretching. This stage precedes the process of necking in the SCJ and its breakup into separate nongradient elements. In the model, the SCJ element is assumed to be an incompressible cylindrical inelastic-perfectly-plastic bar which stretches at a constant value of the Lagrangian axial-velocity gradient. This model [1] is one of the simplest models that describe SCJ stretching in free flight. Nevertheless, it allows one to obtain a rough idea of the kinematic, dynamic, and energetic characteristics of the process and to establish bounds for finding the most important quantitative characteristic of this process, namely, the coefficient of ultimate elongation. These ideas can be substantially cleared up if the physicomathematical model is extended so as to incorporate the compressibility and elastoplastic characteristics of the material. In the present paper, two similar physicomathematical models are presented, and the results obtained by means of these models are discussed.

As follows from [1], at the stage of uniform stretching of a SCJ, when its elements retain a near-cylindrical shape, each element can be assumed to be a cylindrical bar all motion and state parameters of which depend only on the radial coordinate r and time t, with the exception of the axial velocity V_z. This peculiarity is conserved, irrespective of whether the material of the bar is considered incompressible and inelastoplastic, as in [1], or compressible and elastoplastic. Therefore, the process of deformation of each SCJ element in one of its plane cross sections can be treated on the basis of a solution of the one-dimensional unsteady axisymmetric problem of the dynamics of a compressible elastoplastic medium.

The necessity of transition from a plane deformed state in the absence of axial deformations to a triaxial deformed state is a fundamental difference in the formulation of such a problem from similar one-dimensional problems with axial symmetry [2, 3]. A triaxial deformed state occurs in stretching of a cylindrical bar with constant Lagrangian gradient of the axial velocity. The law of conservation of mass must also be changed so as to take into account the decrease in the cross-sectional area of the stretching SCJ element per unit length of the element mass. This can be easily done by the kinematic relations in [1].

The system which governs deformation of a compressible elastic-perfectly-plastic cylindrical stretching bar is of the form

$$dm = 2\pi r\, dr \rho = dm_0/(1 + \dot{\varepsilon}_{z0} t); \tag{1}$$

$$\dot{V}_r = \partial(r\sigma_r)/\partial m - \sigma_\theta/(\rho r); \tag{2}$$

$$\dot{E} = (\sigma_r \dot{\varepsilon}_r + \sigma_\theta \dot{\varepsilon}_\theta + \sigma_z \dot{\varepsilon}_z)/\rho; \tag{3}$$

$$\dot{\varepsilon}_r = \partial V_r/\partial r, \qquad \dot{\varepsilon}_\theta = V_r/r, \qquad \dot{\varepsilon}_z = \dot{\varepsilon}_{z0}/(1 + \dot{\varepsilon}_{z0} t), \qquad \dot{r} = V_r; \tag{4}$$

$$p = K(\rho/\rho_0 - 1)(\rho/\rho_0)^2 + \gamma\rho(E - E_0); \tag{5}$$

$$\dot{\varepsilon}_i = \dot{\varepsilon}_i^e + \dot{\varepsilon}_i^p; \tag{6}$$

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$$\dot{S}_i = 2G[\dot{\varepsilon}_i^e + (1/3)\dot{\rho}/\rho]; \quad (7)$$

$$\dot{\varepsilon}_i^p = \lambda_n S_i; \quad (8)$$

$$S_r^2 + S_\theta^2 + S_z^2 \leqslant (2/3)Y_0^2; \quad (9)$$

$$\sigma_i = -p + S_i. \quad (10)$$

Here m is the radial mass Lagrangian coordinate, ρ is the current density of the material, V_r is the radial component of the velocity vector, E is the specific internal energy; σ_r, σ_θ, and σ_z and S_r, S_θ, and S_z are the radial, tangential, and axial components of the stress tensor and of the stress deviator, respectively; $\dot{\varepsilon}_r$, $\dot{\varepsilon}_\theta$, and $\dot{\varepsilon}_z$ are the corresponding components of the strain-rate tensor, $\dot{\varepsilon}_{z0}$ is the initial axial-velocity gradient [1], p is the pressure; K is the bulk modulus; G is the shear modulus; Y_0 is the yield point of the material, the superscripts e and p denote the elastic and plastic components of the strain-rate tensor, and the coordinate subscript i is assumed to take on any of the following values: $i = r$, θ, and z.

The system includes the law of conservation of mass (1) written for the unit length of an annular element with allowance for its variation during the process of elongation of the bar relative to the initial length in the form $dm_0 = 2\pi r_0\, dr_0 \rho_0$ (r_0 is the radial Lagrangian coordinate of the given annular element), the equation of motion (2), the equation of energy (3) in the adiabatic approximation, and the kinematic relations (4), which also include the axial component of the strain-rate tensor as a function of time [1]. Equations (5)–(9) characterize the physicomechanical characteristics of the SCJ material and include the equation of state (5) and the Prandtl–Rice equations of plastic flow for an ideally elastoplastic material [3, 4]. The latter equations consist of the condition of additivity of the elastic $\dot{\varepsilon}_i^e$ and plastic $\dot{\varepsilon}_i^p$ components of the strain-rate tensor (6), Hooke's law (7) in the differential form for the elastic components of the strain rates, and the associated law of plastic flow (8) in which the scalar factor λ_n was determined by the specific plastic-strain power: $\lambda_n = (3/2)(\dot{\varepsilon}_r^p S_r + \dot{\varepsilon}_\theta^p S_\theta + \dot{\varepsilon}_z^p S_z)/Y_0^2$. The normalizing term E_0 in the equation of state is determined using the standard temperature in proportion to the specific heat of the SCJ material.

We specify the kinematic $V_r(0,t) = 0$ at the bars' axis and dynamic boundary $\sigma_r(R,t) = 0$ conditions at the free lateral surface of the bar $r = R$ as conditions that model the stretching conditions for the SCJ element.

As for the specification of the initial conditions that are adequate to the initial conditions of deformation of SCJ elements immediately after their formation by collapse of the shaped-charge lining, we can reasonably specify the initial axial-velocity gradient $\dot{\varepsilon}_{z0}$ (using the experimentally or numerically determined velocity distribution over the SCJ length) and also the initial specific internal energy E (via the specific heat of the material, which can be found using the initial temperature of the SCJ elements [5]).

Experimental information on the initial values of the other parameters of motion and state of the SCJ elements (radial velocities V_r of motion and the stress-strain characteristics of the material) is lacking, and it is unlikely that this information can be gained by direct measurements at all, because in practice there is no access to the SCJ-formation zone. The necessary initial values of the parameters can be specified, however, using the results obtained within the framework of the model of an incompressible inelastoplastic high-gradient bar, which was proposed in [1]. Calculations have shown that, in this case, the least "stringent" deformation conditions of the SCJ element are predicted, and evaluation of the evolution of their kinematic and stress-strain characteristics is a minimum evaluation.

Under these initial and boundary conditions, system (1)–(10) is solved by numerical integration of the finite-difference scheme [3, 6].

As an example which gives an idea of the qualitative aspect of the SCJ's element stretching process within the framework of the model of a cylindrical compressible elastoplastic bar, Fig. 1 shows calculation results for an element of the middle section of a laboratory copper SCJ with initial radius $R_0 = 3.5$ mm for $\dot{\varepsilon}_{z0} = 3.18 \cdot 10^5$ sec^{-1}, $Y_0 = 0.2$ GPa, $G = 45.6$ GPa, and $K = 80.2$ GPa. The results are represented by the pressure at the axis of the element $\bar{p}_0 = p(0,t)/(\rho_0 \dot{\varepsilon}_{z0}^2 R_0^2)$ (Fig. 1b) and by the radial motion velocity of its lateral surface $\overline{V}_R = V_r(R,t)/(\dot{\varepsilon}_{z0} R_0)$ (Fig. 1a) versus the current coefficient of elongation of the SCJ element $n = 1 + \dot{\varepsilon}_{z0} t$ [1].

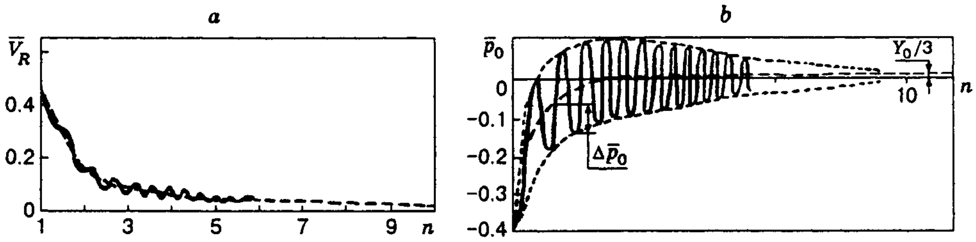

Fig. 1

With allowance for the compressibility and elastoplastic characteristics of the material, SCJ's element stretching calculations predict the existence of an oscillating process. The stress and velocity variations are of an oscillatory character, with fluctuations of the parameters occurring relative to the values (shown by dashed curves) typical of the corresponding incompressible inelastoplastic bar [1].

It is seen from Fig. 1b that, for the incompressible elongating bar, the pressure at the axis of symmetry tends asymptotically to $-Y_0/3$, which is typical of uniaxial stretching. In practice, this value is reached after a fourfold elongation ($n = 4$) of the SCJ element with the above parameters. The existence of the oscillating process, however, results in the fact that at the corresponding moment of time, the stresses act inside the bar which periodically change their sign and have a maximum absolute value of approximately $p_0 = 1$ GPa. The amplitude Δp_0 of the alternating stresses inside the SCJ element decreases with time and eventually approaches a uniaxial-stretching state. The velocity and pressure fluctuations have a variable period T, which decreases uniformly as the bar elongates. In this case, variations in the pressure and in the stress-tensor components along the radial coordinate r during one cycle are smooth. The nearly parabolic character of the distributions is preserved, thus making it possible to characterize the evolution of the stress state during stretching of the bar by the pressure dependence at the axis of symmetry shown in Fig. 1b.

Evidently, the manifestation of the oscillating process during stretching of the SCJ element is due to the compressibility and inertial characteristics inherent in actual materials and taken into account within the framework of the model. The initial density ρ_0 and the bulk modulus K are the quantitative characteristics of compressibility and inertia, respectively. The appearance of the oscillating process and its existence can be explained in the following way.

Stretching of any SCJ element is unsteady. The values of the motion and state parameters of the element, which are determined under the assumption of incompressibility of the material [1], are certain quasi-equilibrium values. For a stretching bar, these values satisfy the laws of conservation of mass and momentum and can be regarded as dynamic analogs of the parameters of rest in a free nonstretching bar.

As noted above, the initial conditions in SCJ's element stretching calculations were given from the incompressibility condition of the material as kinematically and dynamically consistent conditions, in accordance with the model proposed in [1]. Therefore, at the initial time ($t = 0$ and $n = 1$), the values of the motion and state parameters are as if balanced: to the initial axial-velocity gradient $\dot{\varepsilon}_{z0}$ corresponds the linear distribution of the radial velocity V_r over the radius of the element and the parabolic distributions of the stress-tensor components and of the pressure p (Fig. 1).

The model of the incompressibility of a medium implies an infinitely large propagation velocity of any perturbations, so that, as the bar elongates with time and the axial-velocity gradient decreases gradually, the radial motion velocities of the bar particles and the stresses inside the bar (dashed curves in Fig. 1) decrease monotonically. However, real media and the more realistic model of a compressible medium have no such characteristic. If we assume that at a certain moment (for example, at the initial moment), the radial velocity of the outer surface V_R corresponds to the velocity of the incompressible bar, then as early as the next moment this velocity must decrease in magnitude.

Since the SCJ material is inertial, such a decrease in velocity lags in time, and the radial motion converging to the axis continues at a somewhat greater speed, thus leading to the appearance of "surplus" compressive radial stresses (and, consequently, axial stresses and pressure). The latter retard the radial

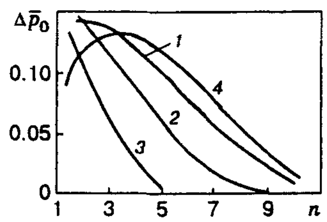

Fig. 2

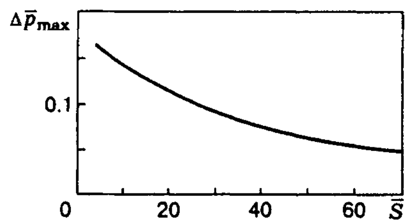

Fig. 3

motion of the jet particles, and, owing to the inertia, the state of "dynamic equilibrium," which is typical of an incompressible bar, turns out again to be passed, this time with decreasing modulus of the radial velocity, thereby resulting in the appearance of tensile stresses, which are "surplus" with respect to the quasi-equilibrium values, etc. This shows that the oscillating process occurs with alternating "surplus" extension and compression phases in the material.

Calculations carried out within the framework of the previously given physicomathematical model of the process of stretching of the SCJ element as a cylindrical compressible elastic-perfectly-plastic bar have shown that the values of its parameters are greatly affected by the initial radius R_0 of the SCJ element, the initial axial-velocity gradient $\dot{\varepsilon}_{z0}$, the density of the material ρ_0, the bulk modulus K, and the yield point Y_0. The shear modulus G of the material has practically no effect on the process of uniform stretching, which corresponds to an insignificant influence of elastic deformations for developed plastic flow during SCJ deformation. Nor do the Grüneisen coefficient γ and the initial temperature of the SCJ material, which corresponds to a weak manifestation of compressibility during SCJ deformation when the density variations and the pressures occurring in the material are not large. In accordance with the theory of dimensionality [7], the nondimensional value of the amplitude of pressure fluctuations at the axis of the bar $\Delta\bar{p}_0 = \Delta p_0/(\rho_0\dot{\varepsilon}_{z0}^2 R_0^2)$ depends on the following nondimensional complexes:

$$\bar{S} = K/(\rho_0\dot{\varepsilon}_{z0}^2 R_0^2), \qquad \bar{U} = Y_0/(\rho_0\dot{\varepsilon}_{z0}^2 R_0^2), \qquad n = 1 + \dot{\varepsilon}_{z0}t. \tag{11}$$

Here the complex $\bar{S}$ can be treated as a complex that characterizes the relative magnitude of the compressibility and inertia of the material of the SCJ element (in accordance with [1], the complex $\rho_0\dot{\varepsilon}_{z0}^2 R_0^2$ is the initial specific kinetic energy of the radial motion of the element material), and the complex $\bar{U}$ can be treated as a complex that characterizes the inertia and the relationship between the level of internal forces arising in the SCJ element in plastic flow [1].

Figures 2 and 3 show the effect of the nondimensional complexes (11) on the nondimensional amplitude $\Delta\bar{p}_0$ of stress oscillations. Figure 2 is based on calculation results at constant $\bar{S}$ and at several values of $\bar{U}$ (curves 1–3 refer to $\bar{U} = 0.018, 0.073$, and 0.29). It is seen from Fig. 2 that, in practice, the maximum amplitude $\Delta\bar{p}_{\max}$ of stress oscillations for the whole process does not depend on the complex $\bar{U}$ and, consequently, is determined by the relation between compressibility and inertia.

Figure 3 is based on calculation results with constant $\bar{U}$ and shows $\Delta\bar{p}_{\max}$ versus the complex $\bar{S}$. The more compressible and denser the SCJ material and the more massive the gradient SCJ element, the greater the amplitude of oscillations. The relation between the plastic forces and the inertia determines the character of oscillation attenuation. The more intense the attenuation, the more significant the factor of dissipation, which is characterized by the quantity Y_0 compared with the "stock of inertia" $\rho_0\dot{\varepsilon}_{z0}^2 R_0^2$.

As for the influence of the governing nondimensional complexes (11) on the oscillation period, the results of numerical calculations of the nondimensional oscillation period $\bar{T} = T\dot{\varepsilon}_{z0}$ are accurately approximated by the following function of only two complexes: $\bar{T} = 2.7/\sqrt{n\bar{S}}$. In dimensional form, the oscillation period is as follows:

$$T = 2.7(R_0/\sqrt{n})\sqrt{\rho_0/K} = 2.7\sqrt{\rho_0 R^2/K}, \tag{12}$$

where R is the current radius of the bar (of the SCJ element). As (12) shows, the oscillation period of the parameters during stretching of the compressible bar is determined by the relation between its mass $\rho_0 R^2$ per unit length and the volume-elasticity characteristic K. Therefore, relation (12) turns out to be quite similar to the relation which is known for the simplest oscillating system with the inertial and elastic elements.

Thus, with allowance for the compressibility and elastoplastic characteristics of the material, simulation of the process of uniform SCJ stretching makes it possible to get additional information on the evolution of the stress-strain state in jet elements. The results obtained by the above model [for example, relation (12) for the oscillation period T] can be treated as "computational," whereas the reasons for the existence of precisely this character of the relationship remain unclear, and, hence, a more comprehensive consideration is required. A similar consideration can be performed using one more model that combines the advantages of the model of [1] for the SCJ element as an incompressible inelastoplastic bar (relative simplicity and the possibility of obtaining the components of an analytical solution) and of the model for the SCJ element as a compressible bar. For this purpose, the model of [1] can be used as a basis, and simplifying assumptions can be based on the results of the more complicated model presented above with retention of the main advantage of the preceding model — allowance for the compressibility of the material.

We assume that in stretching the SCJ element as a compressible inelastoplastic bar, the distribution of the pressure p over the bar radius r has the following parabolic form:

$$p = C_2 r^2 + C_0, \tag{13}$$

where the coefficients C_2 and C_0 are functions of time t alone.

Let us write the law of compressibility of the bar material in the form

$$p = K \ln(\rho/\rho_0), \tag{14}$$

which may well replace the more general equation of state (5) for relatively small pressures and volume deformations realizable in the SCJ material during its stretching, taking into account the weak effect of the thermal part of (5) on the parameters of the oscillating process.

One can find the distribution of the radial component V_r of the particle-velocity vector over the radius r from the solution of the equation of continuity

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \dot{\varepsilon}_z = 0,$$

in which the axial strain rate $\dot{\varepsilon}_z$ is assumed to be a known function of time in accordance with (4). Under assumptions (13) and (14), the equation of continuity is reduced to the form

$$\frac{1}{K}(\dot{C}_2 r^2 + \dot{C}_0 + 2C_2 r V_r) + \frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \dot{\varepsilon}_z = 0.$$

Solving this differential equation with allowance for the boundary condition $V_r = 0$ for $r = 0$, we find

$$V_r = \frac{K}{2C_2}\left(\dot{\varepsilon}_z + \frac{\dot{C}_0}{K} - \frac{\dot{C}_2}{C_2}\right)\left[\exp\left(-\frac{C_2 r^2}{K}\right) - 1\right]\frac{1}{r} - \frac{\dot{C}_2}{2C_2}r.$$

Recalling that the pressure in the inertially deforming SCJ is much lower than the bulk-compression modulus ($C_2 r^2/K \ll 1$), expanding the exponential function into a series we obtain the following expression:

$$V_r = -(\dot{\varepsilon}_z + \dot{C}_0/K)(r/2) - \dot{C}_2 r^3/(4K). \tag{15}$$

To find the unknown functions C_0 and C_2 in the pressure distribution (13), we use the equation of motion (2) for the radial velocity component in the form

$$\rho\frac{dV_r}{dt} = \frac{\partial \sigma_r}{\partial r} + \frac{S_r - S_\theta}{r}.$$

With allowance for the boundary condition $\sigma_r(R) = 0$, integrating this equation over the radius of the bar from a certain current value of r to the outer-surface radius R, we obtain

$$\int_r^R \rho\frac{dV_r}{dt}\,dr = -\sigma_r(r) + \int_r^R \frac{S_r - S_\theta}{r}\,dr,$$

from which follow two relations which link the desired functions C_0 and C_2:

$$C_2R^2 + C_0 - S_r(R) = 0; \tag{16}$$

$$\int_0^R \rho \frac{dV_r}{dt}\, dr = C_0 - S_r(0) + \int_0^R \frac{S_r - S_\theta}{r}\, dr. \tag{17}$$

Owing to the experimentally revealed fact that the effect of elasticity of the SCJ material on the motion and state parameters on stretching is insignificant, to obtain an approximate solution the values of the radial components of the stress deviator at the axis and surface of the bar can be assumed to correspond to those found by the model of an incompressible inelastoplastic bar [1]: $S_r(0) = S_r(R) = -Y_0/3$. To determine approximately the integral on the right-hand side of (17), one can use the Saint Venant–Levy–von Mises relations $S_{ij} = (2/3)(Y_0/\dot{\varepsilon}_i)\dot{\varepsilon}_{ij}$ [4] in which, as for the incompressible bar, the strain-rate intensity $\dot{\varepsilon}_i$ can be assumed to be equal to the current axial strain-rate $\dot{\varepsilon}_i = \dot{\varepsilon}_z$, and the components $\dot{\varepsilon}_{ij}$ of the strain-rate tensor can be found by means of the kinematic relations with the use of the velocity distribution (15):

$$\dot{\varepsilon}_r = \frac{\partial V_r}{\partial r} = -\frac{1}{2}\Big(\dot{\varepsilon}_z + \frac{\dot{C}_0}{K}\Big) - \frac{3}{4}\frac{\dot{C}_2 r^2}{K}, \qquad \dot{\varepsilon}_\theta = \frac{V_r}{r} = -\frac{1}{2}\Big(\dot{\varepsilon}_z + \frac{\dot{C}_0}{K}\Big) - \frac{\dot{C}_2 r^2}{4K}.$$

Under these assumptions, the integral on the right-hand side of (17) is reduced to the following expression:

$$\int_0^R \frac{S_r - S_\theta}{r}\, dr = -\frac{1}{6}\frac{Y_0}{\dot{\varepsilon}_z}\frac{\dot{C}_2 r^2}{K}.$$

Next, determining dV_r/dt via differentiation relations (15), calculating the integral on the left-hand side of (17), assuming, as before, that p is small in the material compared with K, and writing relation (15) for the lateral surface of the bar, we obtain a system of three equations describing the process of stretching of a cylindrical compressible plastic bar:

$$C_2R^2 + C_0 = -Y_0/3; \tag{18}$$

$$\frac{\rho_0 R^2}{4}\Big\{\frac{1}{2}\Big(\dot{\varepsilon}_z + \frac{\dot{C}_0}{K}\Big)^2 + \frac{\dot{C}_2 R^2}{2K}\Big(\dot{\varepsilon}_z + \frac{\dot{C}_0}{K}\Big) + \frac{\dot{C}_2^2 R^4}{8K^2} - \frac{1}{K}\ddot{C}_0 - \frac{R^2}{4K}\ddot{C}_2 - \ddot{\varepsilon}_z\Big\} = C_0 + \frac{Y_0}{3} - \frac{1}{6}\frac{Y_0}{\dot{\varepsilon}_z}\frac{\dot{C}_2 r^2}{K}; \tag{19}$$

$$\frac{dR}{dt} = -\Big(\dot{\varepsilon}_z + \frac{\dot{C}_0}{K}\Big)\frac{R}{2} - \frac{\dot{C}_2 R^3}{4K}. \tag{20}$$

Equations (19) and (20) are ordinary differential equations, and system (18)–(20) is, on the whole, much less complicated than system (1)–(10). In addition to the evident technical advantages that this system offers in gaining numerical information on the process considered, system (18)–(20) enables us to derive, by further transformations, an equation for the forced-oscillation process in the stretching bar (SCJ element) and to establish the analytical dependence of the oscillation period on the SCJ parameters and on the material characteristics.

After elimination of the unknown function C_0, Eqs. (19) and (20) are reduced to the form

$$\ddot{C}_2 + \Big(\frac{8}{9}\frac{Y_0}{\rho_0 \dot{\varepsilon}_z R^2} - \frac{10}{3}\dot{\varepsilon}_z\Big)\dot{C}_2 + \frac{5}{6}\frac{\dot{C}_2^2 R^2}{K} + \frac{16}{3}\frac{K}{\rho_0 R^2} C_2 = \frac{2}{3}\frac{K}{R^2}(2\ddot{\varepsilon}_z - \dot{\varepsilon}_z^2); \tag{21}$$

$$\frac{dR}{dt} = \Big(-\dot{\varepsilon}_z + \frac{\dot{C}_2 R^2}{2K}\Big)\frac{R}{2}. \tag{22}$$

The compressibility effects on variation in the transverse dimensions of the bar can be ignored and, therefore, for the model of an incompressible medium, the current radius of the SCJ element can be assumed to obey the law $R = R_0/\sqrt{1 + \dot{\varepsilon}_{z0} t}$. In this case, relation (21) can be considered irrespective of Eq. (22), which will be used only to determine the velocity V_R of the bar surface.

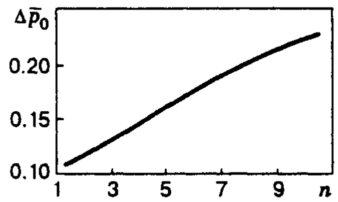

Fig. 4

Thus, we have described the process of dynamic stretching of a compressible plastic bar using, in essence, the second-order ordinary differential equation (21). One can easily see that this equation corresponds to a forced nonlinear oscillating process. Although it cannot be solved analytically, some conclusions on the character of the oscillating process can be made from analysis of the coefficients. For example, the oscillation frequency ω is determined by the coefficient of the desired function $C_2(t)$ itself. The oscillation period is

$$T = 2\pi/\omega = (\pi\sqrt{3}/2)\, R\sqrt{\rho_0/K} = 2.72R\sqrt{\rho_0/K},$$

which coincides almost exactly with relation (12) obtained as an approximation of the results of the computational experiment within the framework of the one-dimensional unsteady problem of the dynamics of a compressible elastoplastic medium and, in fact, explains why the oscillations period is determined by such a relation.

It is known from the theory of oscillations that the coefficient of the first derivative $\dot{C}_2$ in Eq. (22) governs the amplitude of the oscillating process. The oscillation amplitude must decrease with time if the coefficient is positive and must increase otherwise. From the relation between the current and initial values of the radius and the axial-velocity gradient, it follows that the oscillation amplitude increases if the inequalities

$$1 \leqslant n \leqslant \sqrt[3]{(15/4)(\rho_0\dot{\varepsilon}_{z0}R_0^2/Y_0)} \tag{23}$$

hold. This points to the possibility of increasing the oscillation amplitude at the initial stage of stretching if the ratio between the inertial and plastic forces is rather large and also explains the character of the initial sections of the numerical curves plotted in Fig. 2.

In accordance with the simplified model of stretching of an SCJ element, the character of variation of the oscillation amplitude $\Delta\bar{p}$ is determined by numerical integration of Eq. (21). In this case, as initial conditions, we can specify the coefficient C_2 and its time derivative $\dot{C}_2$ determined by the relations of the model of an incompressible inelastoplastic bar [1]: $C_{20} = -3\rho_0\dot{\varepsilon}_{z0}^2/8$ and $\dot{C}_{20} = 3\rho_0\dot{\varepsilon}_{z0}^3/4$.

The calculation of stretching of a compressible inelastoplastic bar is illustrated by curve 4 in Fig. 2. The characteristics of the material and the parameters of the jet element are the same as in the calculation of stretching of a compressible elastoplastic bar shown by curve 1 in Fig. 2. It is seen that the results obtained by the different models are in satisfactory agreement, and this allows us to regard this agreement as an additional test for validity of these models.

In conclusion, we present some practical reasoning following from the specific features of the SCJ behavior that were revealed by physicomathematical simulation during its uniform stretching.

It follows from [1, 8] that the duration of the stage of uniform SCJ elongation and the coefficient of ultimate elongation n_{ult} increase with softening of the material. This corresponds to a decrease in the value of the yield point Y_0 of the SCJ material. In particular, with the material completely softened ($Y_0 = 0$), the ability of the SCJ elements to elongate is unlimited ($n_{\text{ult}} \to \infty$). Some practical procedures of controlling the SCJ-elongation process are required to increase its effective length. The existence of the oscillating process and the character of its evolution under uniform stretching impose, however, certain restrictions on the possibilities of a similar action on SCJ. For example, it follows from (23) that, for the completely softened SCJ material. the oscillation amplitudes of the stresses and of the velocities increase upon stretching of the SCJ.

A similar example is shown in Fig. 4 where calculation results are presented on stretching of a completely softened element of a copper SCJ with the parameters corresponding to curves 1 and 4 in Fig. 2. It is clear that such an increase in the amplitude of stresses acting inside the SCJ can lead to volume failure of its material in tensile-stress phases with subsequent radial scatter of material and with a decrease in the average SCJ density. This will not fail to have a great effect on the through-piercing action of the jet.

Possibly, precisely this circumstance can explain the known character of fracture of lead SCJ — uniform elongation with subsequent "abrupt" volume failure [1]. Lead is very dense, easily compressible, low-melting, and is apparently completely softened under the SCJ conditions. All these factors intensify the oscillating process in stretching a lead SCJ and finally give rise to volume failure. The low penetrative power of lead-lined shaped charges, which, at first glance, contradicts the hydrodynamic theory of cumulation [2], is actually a consequence of a "catastrophe" that occurred with the SCJ at the stage of elongation.

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